Deadbeat Direct Torque and Flux Control of Interior Permanent Magnet Machines with Discrete Time Stator Current and Stator Flux Linkage Observer

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Abstract—This paper presents a discrete time deadbeat-direct torque and flux controller (DB-DTFC) for interior permanent magnet synchronous machines (IPMSMs). A Gopinath-style discrete time flux linkage observer is developed which contains two different flux estimation methods based on current and voltage models for flux linkage. This observer produces correctly estimated flux linkages needed for accurate DB-DTFC voltage models for flux linkage. This observer produces correctly estimated flux linkages needed for accurate DB-DTFC voltage models for flux linkage. It is shown that deadbeats, current and flux linkage observers, the correct single time step (deadbeat) air-gap torque and stator flux linkage control at the (constant) switching frequency is achieved and experimentally evaluated.

Index Terms—Deadbeat control, current and flux linkage observer, interior permanent magnet machine, direct torque control

I. INTRODUCTION

Current vector control (CVC) is the most widely used approach to manipulate the air gap torque of interior permanent magnet synchronous machines (IPMSMs) [1-4]. For CVC, closed loop control of the stator current vector (d-q components) is implemented with the inverter voltage vector being the sole manipulated input. It should be noted, that when using CVC, both air gap torque and flux linkage are open loop variables. Open loop air gap torque dynamics (for the magnet and reluctance torque) are limited by the stator current dynamics but flux linkage has intrinsic first order dynamics.

Bus voltage and current limits affect the stator current dynamics and thus degrade the torque and flux linkage dynamics. In general, current regulation is severely compromised when operating at or near the voltage limits and control laws are modified to deal with these limits.

Since IPMSM torque is comprised of both magnet torque (a vector cross product Lorentz force used for induction machine field oriented control (IM FOC) [5]) and reluctance torque (a vector dot product as used in SR motor drives [6]), CVC-based torque manipulation is not a simple extension of field oriented control. For IPMSMs, the current vector command for CVC is generally not linearly related to torque or flux. Instead, the current vector components are those computed to theoretically yield minimum losses while still achieving the desired total torque. The loss minimization can be based on copper losses, iron losses, inverter losses, or some combination. For example, the commonly used "maximum torque-per-ampere" method [1-4] computes the theoretical current vector to minimize copper losses. Such "optimal" current vector computations can be implemented with varying degrees of complexity, in which saturation and parameter adaptation can be included to improve the optimization [7].

Direct torque control (DTC) is an alternative control structure, initially developed for induction machine (IM) drives [8,9], but more recently investigated also for IPMSMs [10-13], in which closed loop control is applied directly to both the air gap torque and stator flux linkage, using (as in CVC), the inverter voltage vector is the sole manipulated input. Because of the difficulty in decoupling the manipulated inputs (the d-q voltage components) from their effects on the air gap torque and stator flux linkage, bang/bang hysteresis control has been the primary implementation for commercial forms of IM DTC drives [14]. In that case, the inverter switching frequency varies continuously and unpredictably, which can be problematic when inverter losses (and heating) are critical system limits.

For classical DTC, the focus is principally on closed loop torque control dynamics, which are essentially the fastest possible, within the limitations of the inverter voltage. These dynamics are generally faster than CVC torque dynamics, due to the limited dynamics of the closed loop current regulators commonly used for CVC. In DTC, the stator flux linkage command can be manipulated to minimize some combination of losses (similar to the CVC current command calculator), although this is not automatically part of classical DTC.

Since inverter (and device) thermal loading is often critical, a predictable, defined switching frequency can be beneficial. To achieve the torque and flux control advantages of DTC with a constant switching frequency, classical PI control structures have been investigated [13]. Without decoupling the manipulated inputs (the d-q voltage components) this will generally lead to oscillatory dynamics. Thus, an opportunity exits to develop fixed (defined) switching frequency DTC methods that properly deal with the cross-coupled inputs.

This paper focuses on applying deadbeat, direct control of torque and flux (DB-DTFC) using methods similar to those already used successfully for induction machine drives [15-20].
Deadbeat control is a well-established computer control technique in which the inverse (machine) model is solved for the inputs (d-q voltage) that would achieve the desired outputs (air gap torque and stator flux) in just one PWM period, i.e. "dead in one beat". A correctly formed deadbeat control structure inherently decouples the manipulated inputs. For IM drives, this DB-DTFC has been shown to be simple to implement and quite insensitive to parameter errors [21]. In addition, the same control law can be used in the voltage limits, unlike CVC methods, where the current regulator has to be modified to deal with limited bus voltages.

This paper proposes and develops a feasible DB-DTFC solution for IPMSMs, which will both achieve the desired performance and allow a very explicit understanding of how it operates using the same control law both in normal operation and in voltage limits. Experimental results using the proposed DB-DTFC on a 1.5 KW IPMSM drive are used to evaluate the performance and demonstrate how the desired properties have been achieved.

II. DEADBEAT DIRECT TORQUE AND FLUX CONTROL

In this paper, machine parameters and notations are defined as follows:

- \( r_s \) stator resistance
- \( L_d \) d-axis stator inductance
- \( L_q \) q-axis stator inductance
- \( \lambda_{pm} \) permanent-magnet flux linkage
- \( i_{ds}^r \) d-axis stator current in the rotor frame
- \( i_{qs}^r \) q-axis stator current in the rotor frame
- \( v_{ds}^r \) d-axis stator voltage in the rotor frame
- \( v_{qs}^r \) q-axis stator voltage in the rotor frame
- \( \omega_r \) rotor speed
- \( T_s \) PWM (sample) time period

The permanent magnet synchronous machine equations in the rotor reference frame can be written as (1-1) and (1-2) using current components. The equations also can be rewritten as the Faraday's law stator flux linkage differential equation by (2) [22]. The equation is written using d-q complex vector notation where \( f_{dq} = f_d + jf_q \) [5].

\[
\begin{align*}
  v_{ds}^r &= r_s i_{ds}^r + L_d \frac{di_{ds}^r}{dt} + \omega_r L_{qs} i_{qs}^r \quad (1-1) \\
  v_{qs}^r &= r_s i_{qs}^r + L_d \frac{di_{qs}^r}{dt} + \omega_r L_{ds} i_{ds}^r + \omega_r \lambda_{pm} \quad (1-2) \\
  v_{ds}^r &= r_s i_{ds}^r + \frac{d}{dt} \lambda_{ds}^r + j\omega_r \lambda_{qs}^r \quad (2)
\end{align*}
\]

Where \( \lambda_{ds}^r = L_d i_{ds}^r + \omega_r \lambda_{pm} \) and \( \lambda_{qs}^r = L_q i_{qs}^r \).

The air-gap torque equations are developed as shown in (3-1) in the form of the stator current vectors, which separates the reluctance and the magnet torque. The torque equation using the complex vector form of the stator flux linkage is shown in (3-2). In (3-2), \( \cdot \) denotes a dot product (inner product).

\[
T_{em} = \frac{3P}{4} \left( (L_d - L_q) i_{ds}^r i_{qs}^r + \lambda_{pm} i_{qs}^r \right) \quad (3-1)
\]
\[
= \frac{3P}{4} \left( \frac{\lambda_{ds}^r - \omega_r \lambda_{qs}^r}{L_s} \right) \quad (3-2)
\]

The discrete time forms of (2) and (3) are shown as in (4) and (5) that are valid for the short time period of typical PWM frequencies:

\[
\lambda_{ds}^r (k+1) = \lambda_{ds}^r (k) + v_{ds}^r (k) T_s
\]
\[
- \frac{(r_s + L_q \omega_r)}{L_s} \lambda_{ds}^r (k) T_s + \frac{r_s}{L_s} \omega_r \lambda_{pm} T_s \quad (4)
\]
\[
T_{em}(k) = \frac{3P}{4} \left( (L_d - L_q) i_{ds}^r (k) i_{qs}^r (k) + \lambda_{pm} i_{qs}^r (k) \right) \quad (5a)
\]
\[
T_{em}(k) = \frac{3P}{4} \left( \frac{\lambda_{ds}^r (k) - \lambda_{qs}^r (k)}{L_s} \right) \quad (5b)
\]

By substituting (4) into (5b) and forming a differential solution, an expression can be developed for the change in torque, (6),

\[
\Delta T_{em}(k) = T_{em}(k+1) - T_{em}(k) \quad (6)
\]

This can be rewritten to show the linear relationship of the d and q-axis stator voltages with a slope and a constant including \( \Delta T_{em}(k) \) as the commanded change in air gap torque, (6).

\[
v_{qs}^r (k) T_s = -\frac{\left( (L_q - L_d) \lambda_{qs}^r (k) \right)}{\left( L_q - L_d \lambda_{ds}^r (k) - L_d \lambda_{pm} \right)} v_{ds}^r (k) T_s
\]
\[
- \frac{L_d L_q}{L_q - L_d} \alpha_r \lambda_{pm} T_s
\]

\[
\frac{4 \Delta T_{em}}{3P} = \frac{\omega_r T_s}{L_d L_q} \left( (L_q - L_d) (\lambda_{ds}^r (k) - \lambda_{qs}^r (k))^2 - L_d \lambda_{qs}^r (k) \lambda_{pm} \right)
\]
\[
- \frac{R T_s}{L_d L_q} \lambda_{qs}^r (k) \left( (L_q - L_d)^2 (\lambda_{ds}^r (k) - \lambda_{pm}) - L_q^2 \lambda_{pm} \right) \quad (7)
\]

Using (7), multiple possible stator voltage vectors can be calculated which achieve the commanded change in air gap torque over the next sample time instant. Each of these solutions would yield deadbeat torque control, but the stator flux linkage would vary in an uncontrolled fashion. To achieve deadbeat flux linkage control, (4) is formed with the approximated discrete time stator flux linkage equation and the stator resistance terms are treated as being negligible. In addition, the cross-coupling term of the stator flux linkage is decoupled. Then, (3) is approximated as

\[
\lambda_{qs}^r (k) = \lambda_{ds}^r (k) + v_{ds}^r (k) T_s \quad (8)
\]

For operating conditions when voltage is not near the limits, multiple stator flux linkage solutions exist. For constant stator flux linkage magnitude (a circular trajectory), the stator flux linkage would be given by (9).

\[
\lambda_{qs}^r (k) = \lambda_{ds}^r (k) + v_{ds}^r (k) T_s
\]

\[
= \left( \lambda_{ds}^r (k) + v_{ds}^r (k) T_s \right)^2 + \left( \lambda_{qs}^r (k) + v_{qs}^r (k) T_s \right)^2 \quad (9)
\]
Fig. 1 shows a graphical form of the stator voltage solution of DB-DTFC for IPMSMs including the desired torque line and a constant stator flux linkage circle following (9).

For this case, the next sample time stator flux linkage magnitude is generated by the stator voltage vectors which fall on the stator flux linkage circle. As seen in Fig. 1, there are two possible stator voltage vectors, \( v_{dqsr}^{(a)} \) and \( v_{dqsr}^{(b)} \), which satisfy (7) and (9). During the experiment \( v_{dqsr}^{(a)} \) was selected because \( v_{dqsr}^{(b)} \) is higher than the bus voltage limitation of the drive test stand.

III. DIGITAL IMPLEMENTATION OF FLUX LINKAGE OBSERVER

Since stator flux linkage is not measurable, modified Gopinath-style flux observers have been developed to estimate stator flux linkage in the continuous time domain [23-27] and in the discrete time domain [28-29] for an induction machine.

In this paper, a discrete time Gopinath-style stator flux linkage observer for IPMSMs is proposed. The stator flux linkage observer combines two different flux linkage estimation methods similar to [25].

The two methods are based on a current model and a voltage model, analogous to the known induction machine flux linkage observers [25].

The first step is to estimate the stator flux linkage through a current model. The current model is developed based on the stator flux linkage and current equations as (10). The current model can be written in the discrete time form as (11).

\[
\lambda_{r ds} = L_d i_{r ds} + \lambda_{pm} \tag{10}
\]

\[
\lambda_{r ds}^{(k+1)} = L_d i_{r ds}^{(k)} + \lambda_{pm} \tag{11}
\]

The second step to estimate the stator flux linkage is a voltage model-based observer development. The stator flux linkage is decoupled to simplify the stator flux linkage observer model and to eliminate effect of the cross-coupling. Then the voltage model in the continuous time can be approximated by (12).

\[
\omega_p (\lambda_{r dq}^{(k+1)} - j \lambda_{r dq}^{(k)})
\]

b. The proposed voltage model of the stator flux linkage observer in discrete time domain

The third step is to implement the DB-DTFC algorithm.
\[
\frac{d}{dt} \lambda_{dqs}^r = v_{dqs}^r - r_s i_{dqs}^r 
\] (12)

When converting (12) from the continuous to discrete time, it should be noted that stator voltage is the only latched manipulated input to the system and the stator current is not a latched input, but rather a ramped signal. Therefore, a latched interface should not be applied to convert from continuous to discrete time. The exact stator flux linkage observer model is very complicated [28]. An approximate solution is proposed as shown in Fig. 2. In Fig. 2.a, the stator voltage is latched (zero-order hold) and an additional integration (ramp) is added for the discrete time model of the stator current (first-order hold). From Fig. 2.b, the discrete time form of the voltage model can be written as (13).

\[
\lambda_{dqs}^r(k+1) - \lambda_{dqs}^r(k) = v_{dqs}^r(k) T_s - \frac{T_s}{2} r_s (i_{dqs}^r(k+1) + i_{dqs}^r(k)) 
\] (13)

The current model is recommended at low frequencies because the voltage model is sensitive to stator resistance as seen in (12). The voltage model is preferred at high frequencies because it has virtually no parameter sensitivity in that region. Therefore, the single closed-loop observer combining the current and voltage models for IPMSMs is a key element in this paper. The combined stator flux linkage observer in the discrete time domain is shown in Fig. 3.

This flux linkage observer has a form similar to a Gopinath-style observer. The transition between the voltage model and the current model is determined by the bandwidth for which the stator flux linkage observer controller is tuned.

The stator flux linkage is not a measurable physical variable. Therefore the proposed stator flux linkage observer is verified by measuring the angle of the estimated stator flux linkage. The stator flux linkage can be divided into d-axis and q-axis, and the angle between the two axes is coincident to the angle information from an encoder which is zero.

\[
\theta_j(k) = \tan^{-1} \left( \frac{\lambda_{qs}^r(k+1)}{\lambda_{ds}^r(k+1)} \right) = \tan^{-1} \left( \frac{\lambda_{qs}^r(k+1) - L_q i_{qs}^r(k+1)}{\lambda_{ds}^r(k+1) - L_p \lambda_{pm}} \right) = \tan^{-1} \left( 0 \right) = 0 
\] (14)

Fig. 4 shows the experimental result of the angle between d and q axes of the estimated stator flux linkage using the stator flux linkage observer.

IV. ROTOR FRAME-BASED STATOR CURRENT OBSERVER

To implement the deadbeat-direct torque control algorithm, the stator current at the next sample instant needs to be estimated. It can be estimated by a stator current observer in the rotor reference frame. The stator current observer, as implemented and experimentally evaluated, is based on the IPMSMs state equations. For the stator flux linkage observer, the cross-coupling is decoupled when the stator current observer is developed.

To form the discrete time stator current observer, a latched stator voltage is applied. The decoupled and latched input discrete time version of IPMSMs model is written as (15).

\[
i_{dqs}^r(k+1) = e^{-T_s \hat{r}} i_{dqs}^r(k) + \frac{v_{dqs}^r(k)}{r_s} (1 - e^{-T_s \hat{r}}) I 
\] (15)

where \( \hat{r} = L_q \frac{L_p}{r_s} \)

The stator current in next sample time instant can be estimated with the discrete time version of the stator current observer. The block diagram of the continuous and discrete time stator current observer for IPMSMs is shown in Fig. 5.

Fig. 6 shows the experimental verification of the rotor reference frame-based stator current observer. The test results show how the next step estimated current leads as expected during transients of the d-q current vector.
V. EXPERIMENTAL RESULTS

The DB-DTFC algorithm for IPMSMs presented in the paper is developed and implemented experimentally. The mechanical characteristics and the parameters of the IPMSMs used during the experiment are arranged in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>10 $\Omega$</td>
</tr>
<tr>
<td>$L_d$</td>
<td>17.5 mH</td>
</tr>
<tr>
<td>$L_q$</td>
<td>23.5 mH</td>
</tr>
<tr>
<td>$\lambda_{pm}$</td>
<td>0.086 Wb</td>
</tr>
<tr>
<td>$J_p$</td>
<td>1.0 kg·m²·x10⁻⁴</td>
</tr>
<tr>
<td>$T_s$</td>
<td>0.0001 (sec)</td>
</tr>
<tr>
<td>Poles</td>
<td>4</td>
</tr>
<tr>
<td>Max. Torque</td>
<td>7.7 Nm</td>
</tr>
<tr>
<td>Max. Speed</td>
<td>6200 rpm</td>
</tr>
</tbody>
</table>

With a functional stator current observer and the stator flux linkage observer, the DB-DTFC system can be implemented. Fig. 7 illustrates the block diagram of the entire control structure showing all portions integrated together including the stator current and the stator flux linkage observers.

Fig. 6. Experimental results of estimation of next sample time stator current using the proposed stator current observer

Fig. 7. Block diagram of the entire DB-DTFC system.
A. Experimental Verification

DB-DTFC is implemented in the inverter with 10 kHz of PWM sampling frequency. An AIX XCS 2000 DSP is used to control three phase inverters consisting of Semikron IGBT six packs rated for 600V and 30 A. Fig. 8 shows the experimental test set up with a surface permanent magnetic synchronous machine (SPMSM) load machine mechanically coupled to the IPMSM test machine.

![Fig. 8. Experimental test set up, Left-SPMSM load and Right IPMSM test](image)

The SPMSM is rated for 1.5 hp and 5,000 rpm maximum speed. The SPMSM is field oriented and speed controlled.

To compare dynamic response and parameter sensitivity of DB-DTFC versus CVC, the CVC algorithm is also implemented. A bandwidth of the CVC tuned to be 500 Hz and the input current command for CVC is generated based on a maximum-torque-per-amperes (MTPA) look-up table.

Fig. 9 shows the overall block diagrams for both the MTPA CVC and the DB-DTFC in the IPMSM test machine.

![Fig. 9. Block diagrams for control systems: a. MTPA-CVC & b. DB-DTFC](image)

To confirm deadbeat properties, dynamic responses are observed. Fig. 10 shows a step change in torque command at 300 μ-sec. The magnitude of the step command was limited to be a feasible command signal of 0.4 Nm.

![Fig. 10. Experimental results of torque response with DB-DTFC and CVC](image)

Experiment Setting:
\[ \omega_r = 200 \text{ [rad/sec]}, \quad T_s = 100 \text{ [μ-sec]} \]

Bandwidth of the current regulator = 500 [Hz]

Fig. 10 shows that the torque response from the DB-DTFC system tracks the commanded torque in a single time step (deadbeat) and it is faster response than that from CVC.

Fig. 11 shows the stator voltage and stator flux linkage dynamics of IPMSMs with the DB-DTFC algorithm when a feasible step command is applied to the system.

![Fig. 11. Experimental results of stator voltage and estimated stator flux linkage using the DB-DTFC algorithm](image)

Experiment Setting:
\[ \omega_r = 200 \text{ [rad/sec]}, \quad T_s = 100 \text{ [μ-sec]} \]

As seen Fig. 11.a the stator voltage almost reaches its maximum voltage at the sample time instant when the step signal is commanded to the system. Fig. 11.a and 11.b show changes in the stator voltage vectors and the stator flux linkage vectors as the commanded torque varies from zero to 0.4 Nm.
### B. Parameter Sensitivity

The dynamic performance of DB-DTFC versus the MTPA-CVC for IPMSMs was also evaluated by measuring the command tracking frequency response. To estimate how the control systems are sensitive to a parameter estimation error, each parameter in the control system was individually detuned by 20%. The varied parameters are stator resistance, d-axis and q-axis stator inductance, and a permanent magnetic flux linkage. A broad spectrum chirp signal was used as the torque command to each system and the system response is compared to the commanded signal to extract magnitude and phase.

Torque command parameter sensitivity is estimated with 200 rad/sec rotor speed and 10 kHz switching/sampling frequency. Fig 12 shows the torque command tracking parameter sensitivity with MTPA-CVC. The magnitude plot of the torque command tracking frequency response uses a linear scale, which helps to focus on parameter sensitivity.

![Fig. 12. Experimental torque command tracking frequency response function with the CVC algorithm](image)

<table>
<thead>
<tr>
<th>( T_{\text{em,cvc}} / T_{\text{em}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency [Hz]</td>
</tr>
<tr>
<td>Phase lag [degree]</td>
</tr>
</tbody>
</table>

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**Experiment Setting:**
- \( \omega_r = 200 \) [rad/sec], \( T_s = 100 \) [\( \mu \)-sec]
- Bandwidth of the current regulator = 500 [Hz]

The general roll-off is consistent with the bandwidth of the current regulator. It is observed that the magnitude tracking is very sensitive to the permanent magnetic flux linkage over the entire frequency range. However, phase error is almost zero with the permanent magnetic flux linkage estimation error. At low frequencies the magnitude and phase error is almost zero with variation of the stator resistance and the stator inductance. The system becomes sensitive to the stator resistance and the stator inductance at high frequencies.

![Fig. 13. Experimental torque command tracking frequency response function with the DB-DTFC algorithm](image)

**Experiment Setting:**
- \( \omega_r = 200 \) [rad/sec], \( T_s = 100 \) [\( \mu \)-sec]

As seen Fig. 13, the magnitude and the phase torque command tracking errors due to parameter estimation are negligible over the entire frequency range. Also, the DB-DTFC has a virtually constant magnitude ratio and a linear phase shift property, both of which are consistent with discrete time deadbeat control design theory.

### VI. CONCLUSIONS

A DB-DTFC control structure and the corresponding discrete time model for IPMSMs are developed mathematically and graphically in the paper.

Digital implementation of stator current and the stator flux linkage observers for IPMSMs is also introduced and used to evaluate the DB-DTFC IPMSM drive. The stator current and the stator flux linkage observers estimate the stator current and the stator flux linkage at the next sample time instant. Experimental results using each observer were presented and generally confirm the estimation desired estimation properties.

The stator current and the stator flux linkage observers were combined to implement the proposed DB-DTFC algorithm for IPMSMs.

The dynamic response of IPMSM drives using DB-DTFC and MTPA-CVC were evaluated on a test stand switching/sampling at 10 kHz. It was demonstrated that torque dynamics
from the DB-DTFC drive system tracks the commanded torque in a single PWM time step (deadbeat).

Finally, parameter sensitivity of the IPMSM drive system with DB-DTFC and with MTPA-CVC was evaluated using command tracking frequency response. It was shown that the system with the DB-DTFC has superior dynamics and is less sensitive to parameter estimation error than the system with MTPA-CVC.

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